vector length

1. The vectors p, q and r are defined as follows:

$$p = 3i - 3j + 2k, \; q = 4i - j + k, \; r = 4i - 2j + 3k.$$

(a) Find $2p - q + r$ in terms of i, j and k. **1**
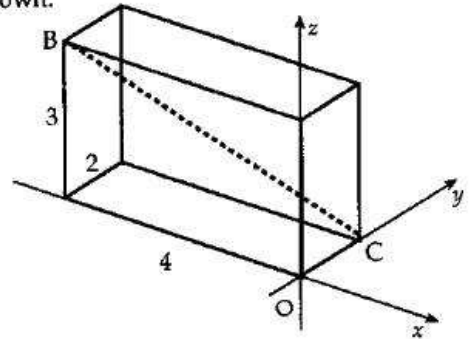
(b) Find the value of $|2p - q + r|$. **2**

2. A cuboid crystal is placed relative to the coordinate axes as shown.

(a) Write down $\overrightarrow{BC}$ in component form.

(b) Calculate $\left|\overrightarrow{BC}\right|$. **2**

3. A is the point $(-3,2,4)$ and B is $(-1,3,2)$. Find
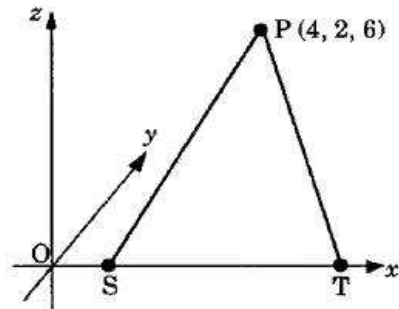
(a) the components of vector $\overrightarrow{AB}$; **1**

(b) the length of AB. **2**

4. The diagram shows a point P with coordinates $(4, 2, 6)$ and two points S and T which lie on the x-axis. If P is 7 units from S and 7 units from T, find the coordinates of S and T. **3**

5. Vectors p, q and r are defined by

$$p = i + j - k, \quad q = i + 4k \quad \text{and} \quad r = 4i - 3j.$$

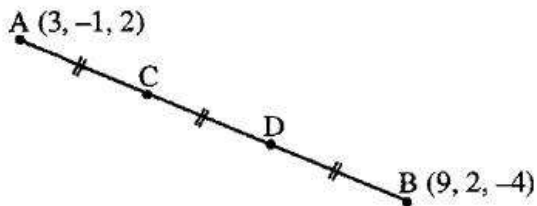(a) Express $p - q + 2r$ in component form. **2**

(b) Calculate $p.r$ **1**

(c) Find $|r|$. **1**

6. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates $(3, -1, 2)$ and $(9, 2, -4)$.
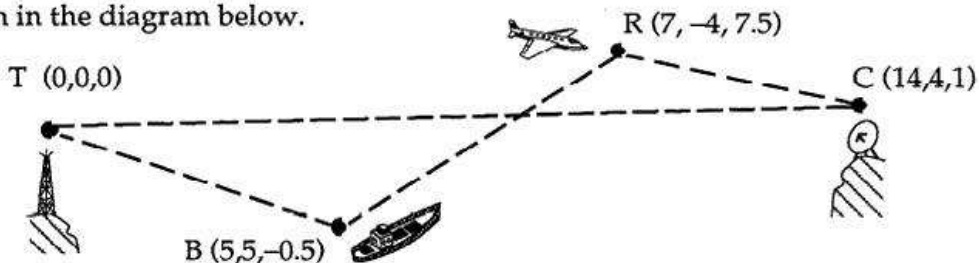
A $(3, -1, 2)$

C

D

B $(9, 2, -4)$

(a) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. 2

(b) Find the coordinates of C and D. 2

7. Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.

R $(7, -4, 7.5)$

T $(0,0,0)$

C $(14,4,1)$

B $(5,5,-0.5)$

The top T of the transmitter mast is the origin, the bridge B on the ship is the point $(5, 5, -0.5)$, the centre C of the dish on the top of a mountain is the point $(14, 4, 1)$ and the reflector R on the aircraft is the point $(7, -4, 7.5)$.

(a) Find the distance from the bridge of the ship to the reflector on the aircraft. (3)

(b) Three minutes earlier the aircraft was at the point M$(-2, 4, 8.5)$. Find the speed of the aircraft in kilometres per hour. (2)

(c) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. (3)

(d) Calculate the size of angle TCR. (5)

8.

With coordinate axes as shown, the point A is (2,4,6).

(a) Write down the coordinates of B,C and D. **(3)**

(b) Show that C is the midpoint of AD. **(1)**

(c) By using the components of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

(d) Hence calculate the size of angle OAB. **(2)**
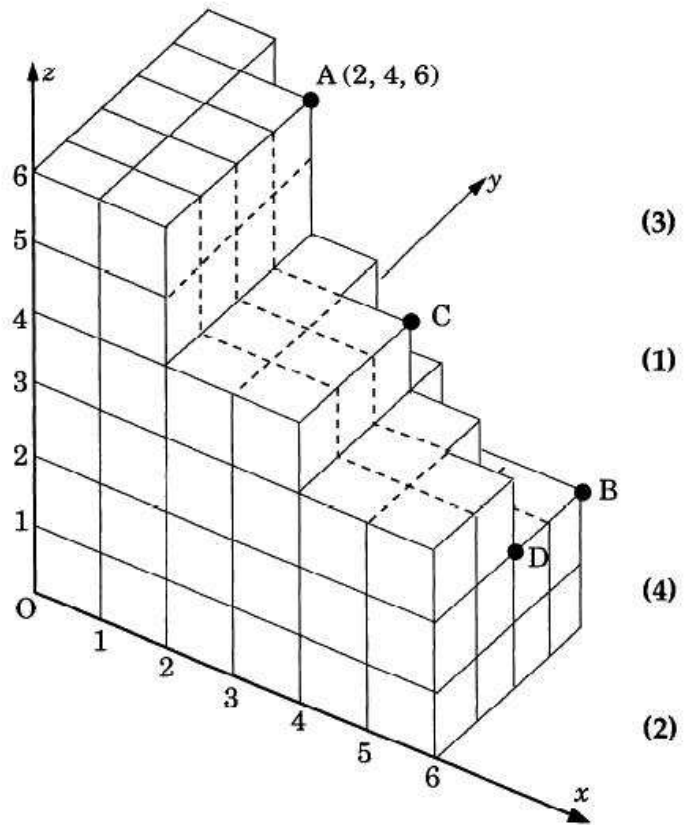
9.

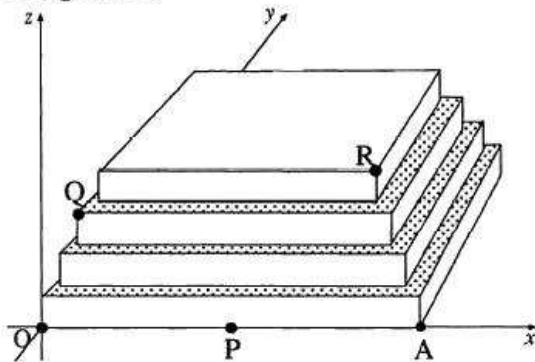The first four levels of a stepped pyramid with a square base are shown in Diagram 1.
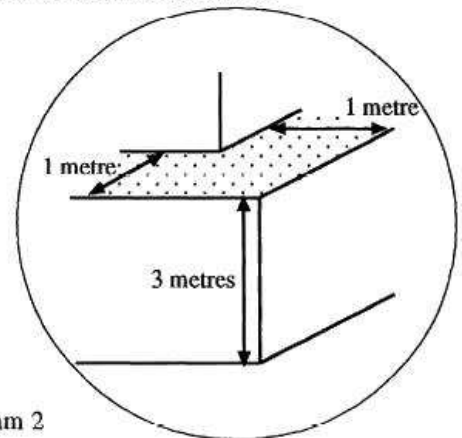
Diagram 1

Diagram 2

Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m.

The height and "width" of a step at a corner are shown in the enlargement in Diagram 2.

With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0).

(a) Find the coordinates of Q and R. **(2)**

(b) Find the size of angle QPR. **(7)**

10.

A triangle ABC has vertices
A $(2, -1, 3)$, B$(3, 6, 5)$ and C $(6, 6, -2)$.

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(b) Calculate the size of angle BAC. **(5)**

(c) Hence find the area of the triangle. **(2)**
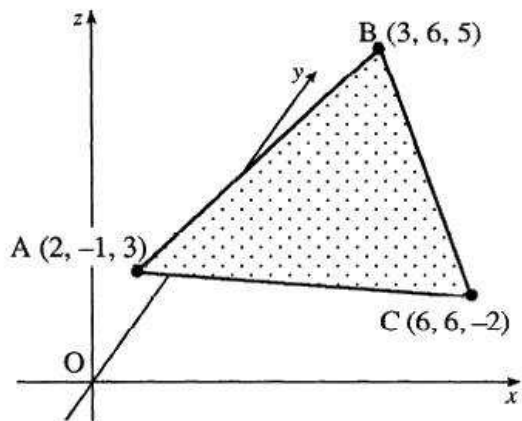
11.

A model of a crystal was made from a cube of side 3 units by slicing off the corner at P to leave a triangular face ABC. Coordinate axes have been introduced as shown in the diagram.
The point A divides OP in the ratio 1:2.
Points B and C similarly divide RP and SP respectively in the ratio 1:2.
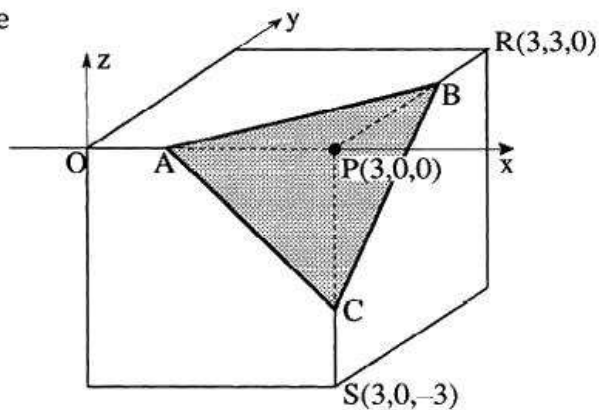
(a) Find the coordinates of A, B and C. **(3)**

(b) Calculate the area of triangle ABC. **(4)**

(c) Calculate the percentage increase or decrease in the surface area of the crystal compared with the cube. **(5)**

12. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.
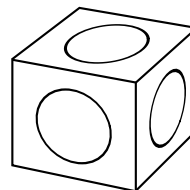
The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are $A(6,0,7)$, $B(0,5,6)$ and $C(4,5,0)$.
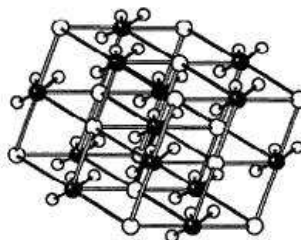
Find the size of angle ABC.

7
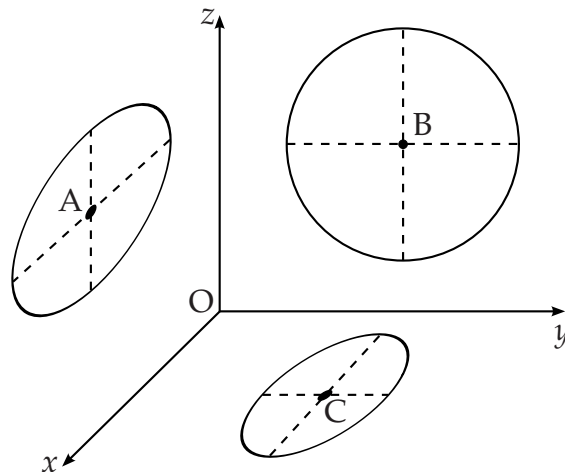
13. The position vectors of the points P and Q are $p = -i + 3j + 4k$ and $q = 7i - j + 5k$ respectively.

(a) Express $\overrightarrow{PQ}$ in component form.   2

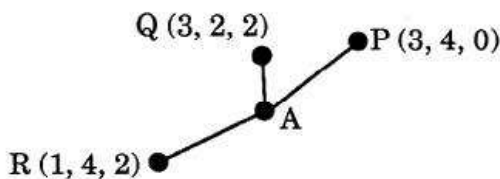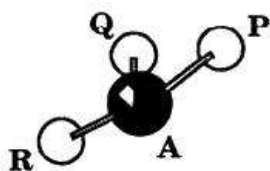(b) Find the length of PQ.   1

14. The diagram shows the rhombohedral crystal lattice of calcium carbonate.

The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown below.

Q (3, 2, 2)   P (3, 4, 0)

A

R (1, 4, 2)

(a) Calculate the size of angle PQR.   (4)

(b) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.

   (i)   Find the coordinates of T.

   (ii) Show that P, Q and R are equidistant from T.   (6)

(c) The coordinates of A are $(2, 3, 1)$.

   (i)   Show that P, Q and R are also equidistant from A

   (ii) Explain why T, and not A, is the centre of the circle through P, Q and R.   (2)

15. Diagram 1 shows a christmas tree decoration which is made of coloured glass rods in the shape of a square-based prism topped by a square pyramid. Diagram 2 shows the decoration relative to the origin and rectangular coordinate axes OX, OY and OZ.

The vertex F has position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$

and the vertex V has position vector $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$
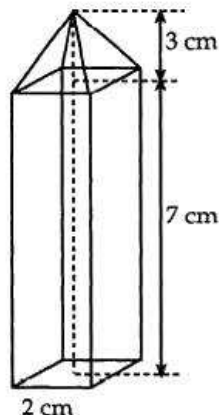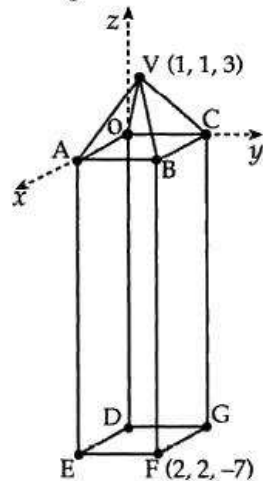

Diagram 1

Diagram 2

(a) Find
   (i) the components of the vectors represented by $\overrightarrow{VF}$ and $\overrightarrow{VE}$;
   (ii) the size of angle EVF. **(7)**
(b) To make the decoration more attractive, triangular sheets of coloured glass VEF and VDG are added to it.
   Calculate the area of the glass triangle VEF. **(3)**

[END OF QUESTIONS]